

A Spatial Spectral Filtration (SSF) Based Correlated Coefficients Thresholding Approach for Image Denoising.

Md Ateeq ur Rahman¹, Abdul Samad Khan²

¹Professor, Department of Computer Science & Engineering,
Shadan College of Engineering & Technology, Hyderabad, India.

²Assistant Professor, Dept of CSE,
JAZAN UNIVERSITY, KSA.

ABSTRACT— In this paper, an improvised dynamic noise filtration technique is proposed for the denoising of images which is based on the filtration of spectral content of the image. This developed approach is termed as Spatial Spectral Filtration (SSF). In this denoising method, a spectral decomposition in multi frequency band using multiwavelets is presented and an enhanced thresholding concept is employed for suppression of the additive noise from the extracted frequency band information.

The proposed method is based on the concept of recovering the spatial dependence of pixels in the noisy image that underwent the multiwavelet decomposition. The resulting decomposed coefficients that are highly correlated are taken as components of a vector and the thresholding operation is applied on the whole vector. In this work we have proposed an enhanced multivariate thresholding scheme which is designed especially for denoising of two dimensional images.

Simulation is performed on images distorted with additive white Gaussian noise at different levels and the obtained results reveal that this method is able to successfully eliminate noise to a reasonable extent and also the performance of this approach significantly surpasses that of conventional denoising techniques both subjectively and visually.

Keywords—Denoising, Gaussian Noise, Multiwavelets, Thresholding, Decomposition.

I. INTRODUCTION

It is found that the image is contaminated with a lot of distortions during its capturing and transmission. These distortions results in noise intensities, blurriness and visual disturbances in the images which in turn leads to major errors in the prediction of bounding regions and estimation of descriptive features of the captured image. Recently, a range of nonlinear median type filtration techniques like weighted median [1] and relaxed median [2] has been proposed for overcoming this drawback. The wiener filtering [3] technique needs the data of the spectra of noise and original signal and is found to filter well only for smooth signals. The wiener filter [3] performs spatial smoothing and its model complexity control depends on selecting the window size. For overcoming the drawbacks of the wiener filtering, the wavelet based denoising approach was proposed in [4].

Filters in wavelet transform processing require a number of desirable features like regularity, symmetry, compact support and orthogonality. However due to implementation constraints, the scalar wavelets [5-7] cannot offer all these features simultaneously leading to less efficient denoising results than multiwavelets [8] which possess all these features simultaneously and offers much efficient processing capabilities than normal wavelets.

A multiwavelet system [8-11] can enhance the performance by offering superior processing at the borders employing linear-phase symmetry, Orthogonality, vanishing moments. Most of the existing methods [8-12] using multiwavelets, works only for one-dimensional signals for denoising of images. The problem in these approaches is that the thresholding technique independently processes the noise on each individual coefficients leading to less accurate denoising results.

For overcoming these drawbacks, an improvised dynamic noise filtration technique is proposed in this paper which is based on the filtration of spectral content of the image. This developed approach is termed as Spatial Spectral Filtration (SSF). In this denoising method, a spectral decomposition in multi frequency band using multiwavelets is presented and a thresholding concept is employed for suppression of the additive noise from the extracted frequency band information.

The proposed method is based on the concept of recovering the spatial dependence of pixels in the noisy image that underwent the multiwavelet decomposition. The resulting decomposed coefficients that are highly correlated are taken as components of a vector and the thresholding operation is applied on the whole vector. In this work we have proposed an enhanced multivariate thresholding scheme which is designed especially for denoising of two dimensional images.

Simulations are performed on images distorted with additive white Gaussian noise at different levels and the obtained results reveal that this method is able to successfully eliminate noise to a reasonable extent and also the performance of this approach significantly surpasses that of conventional denoising techniques both subjectively and visually.

II. THE MULTIWAVELET TRANSFORM AND THRESHOLDING

A. Denoising Process Overview

The process of denoising an image can be mathematically described as follows,

$$y = x + n \tag{1}$$

Where y is the noisy image, x is the original image and n is the AWGN (Additive White Gaussian Noise) with a variance σ^2 . The thresholding technique for denoising of image is intended to eliminate the noise present in the signal without losing original signal characteristics. It consists of the following steps:

1. Acquire the noisy digital signal.
2. Perform the multiwavelet transform of the noisy signal.
3. Compute the threshold value and perform the thresholding of the decomposed noisy signal coefficients.
4. Perform the inverse transform of the thresholded multiwavelet coefficients to get the denoised image.

The above four-step process is called as thresholding as shown is figure 1.

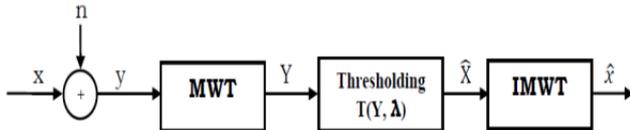


Fig. 1: The three steps of Multiwavelet denoising process with thresholding

Where the thresholding transformation $T(Y, \lambda)$ with the threshold value λ should be such that PSNR (Peak Signal to Noise Ratio) is minimum. Also, it is desirable that the denoised image coefficients \hat{x} obtained after inverse multiwavelet transformation \hat{X} should satisfy conditions like smoothness in low activity regions and sharpness of edges.

B. Pre-Processing

Since the input signal incorporates single stream but the filter bank requires two streams for processing, a technique for providing the data to the two streams has been developed. This technique is called preprocessing and is achieved by a prefilter. A postfilter, at the other end is used to combine data into single stream from multiple streams for image reconstruction. Before using the prefilters, it should be ensured that the required properties of multiwavelets such as orthogonality, short support etc should be maintained as much as possible.

The selection of a prefilter is an important factor for the performance of any specific application and should be carefully chosen. In our simulations, the repeated row prefilter [17] has been used and the obtained results demonstrate that best image denoising performance is achieved. The figure 2 shows the pre-processing process for a single level multiwavelet transformation.

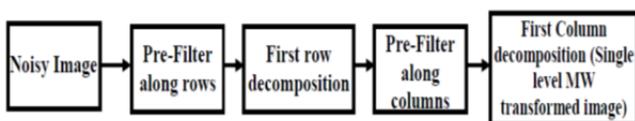


Fig. 2: Pre-Processing process for a single level MW transformation

III. SSF APPROACH IMPLEMENTATION DETAILS

A. Scalar Wavelet Decomposition:

During the processing of a single level decomposition of the image employing scalar wavelet, the two dimensional data is replicated to four blocks. These blocks correspond to subbands where either lowpass filtering or highpass filtering is performed in each direction. In the wavelet decomposition process, the rows and columns of the two dimensional data are subjected to consecutive operations. In this technique, initially the first step transformation is performed on all the rows by the wavelet. This process results in a matrix whose left part includes the down sampled lowpass coefficients of each row, and highpass coefficients are on the right side. In the next step all the columns are subjected to decomposition resulting in the following four types of coefficients.

- (1) HH contains the high-frequency components of the image obtained by performing highpass filtration in both the directions and incorporates the diagonal features.
- (2) HL features are obtained by performing lowpass filtration of rows and then doing the highpass filtration of columns. The horizontal features of the image are included in HL .
- (3) LH incorporates the vertical features of the image and is defined by performing the highpass filtration of rows and then doing the lowpass filtration of columns.
- (4) LL features are obtained by performing lowpass filtration in both the rows and columns. It contains the coefficients that are to be further processed for decomposition in the next level.

The first three subbands, i.e. HL , LH and HH are known as detail subbands as they include high frequency details of the approximation image. The last LL sub-band includes a rough description of the image and contains the output of the low pass filtration along with both rows and columns.

The figure 3 displays the HH subband and the four coefficients will constitute a vector. Then each of these coefficients vectors are processed and subjected to a multivariate thresholding operation. The same thresholding operation is repeated for all the coefficients in all three subbands HH , HL , and LH separately.

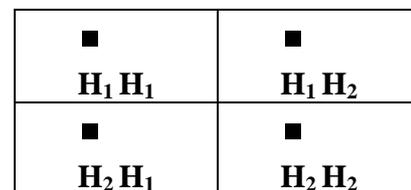


Fig. 3: The HH subband after single level decomposition using multiwavelet

Applying the multiwavelet transformation by prefiltering the noisy image, we are able to obtain the vector coefficients by combining each subband coefficients in vectors of length four as given in equation (2).

$$W_{j,k} = V_{j,k} + \rho_{j,k} \tag{2}$$

Where,

- V represents noise-free multiwavelet coefficient vector,
- ρ points to coefficient vector of multiwavelet for the noise,
- W indicates the coefficient vector of multiwavelet for the corrupted signal,
- j represents the decomposition level, and
- k indicates the coefficient index.
- $\rho_{j,k}$ includes the multivariate normal distribution $N(0, \Theta_j)$ and,
- Θ_j is based on resolution level j and is the covariance matrix of the noise term.

For distributing each coefficient within the vector independently, it is recommended to whiten the noise. The whitening operation can be done by multiplying the noise vector with $\Theta_j^{-1/2}$ by assuming that the vector doesn't have any signal component in it.

If $y_{j,k} = \Theta_j^{-1/2} w_{j,k}$ then it can easily be proved that the covariance matrix of y is the identity matrix. The squared length of the vector y can be estimated by equation (3).

$$\zeta_{j,k} = y_{j,k}^T y_{j,k} = w_{j,k}^T \Theta_j^{-1} w_{j,k} \tag{3}$$

Where T represents the transpose. $\zeta_{j,k}$ represents a positive scalar value with a Chi-squared distribution having four degrees of freedom. The analogous hard thresholding rules can be applied in (4) and the soft thresholding in (5) as given below.

$$\hat{w}_{j,k} = w_{j,k} \cdot 1(\zeta_{j,k} \geq \lambda) \tag{4}$$

$$\hat{w}_{j,k} = w_{j,k} \cdot \frac{\max(\zeta_{j,k} - \lambda, 0)}{\zeta_{j,k}} \tag{5}$$

This thresholding method processes the largely related multiwavelet coefficients together and applies a common threshold to the vector of coefficients. The effect of correlation is minimised by applying the whitening transformation. The covariance matrix employed for the transformation is estimated separately for the subbands HH , HL , and LH for each decomposition level.

B. Threshold Selection:

The choice of threshold parameter is a very important factor in denoising process whose value determines, to a great extent, the efficacy of denoising. For image denoising applications, the soft thresholding technique generally provides more visually pleasing results than hard thresholding, and it is therefore adopted in this paper.

For computing the value of threshold parameter λ , suppose that there are N identically and independently distributed x_4^2 random variables and M represents the maximum of these random variables. The value threshold λ can then be computed as consisting of infimum of all sequences λ_N such that if the multiwavelet transformation is represented by M and preprocessing indicated by P then,

$$P(M \leq \lambda_N) \longrightarrow 1 \text{ as } N \longrightarrow \infty \tag{6}$$

From the above equation, the probability of the threshold value being larger than the maximum of the noise random coefficients reaches to one as number of pixels goes to infinity. This implies that, asymptotically, if the signal component reaches to zero, then the false alarm probability will also lead to zero, as a result of which the combination of zero signal and the noise signal cannot go beyond the threshold level, and therefore has to be put to zero.

This ensures that, with high probability, the coefficients consist of signal component which are greater than threshold value. The threshold value can be estimated by solving for λ and using the CDF (Cumulative Distribution Function) of M in the limit problem (6). From [17] a suitable sequence fulfilling the relation in (6) is given in (7).

$$\lambda_N = \sqrt{(2 \log N + 2 \log \log N)} \tag{7}$$

Where N indicates the number of all high frequency coefficients in wavelet domain. Therefore, using this value the multiwavelet coefficient vectors are thresholded and the noisy coefficients present in the image are eliminated. The threshold value should be adjusted depending on the noise level in the image.

Finally the image reconstruction from the denoised coefficients is done by performing the inverse multiwavelet transform on the thresholded coefficients. The whole process is depicted in block diagram given in figure 4.

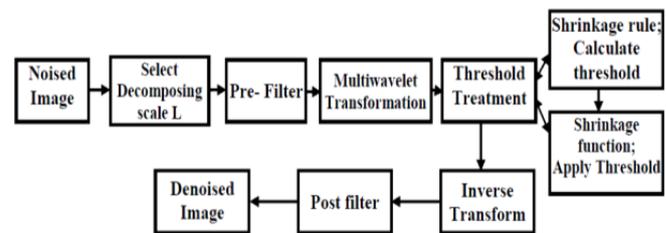


Fig. 4: Block diagram of SSF based image denoising technique

IV. SIMULATION RESULTS AND DISCUSSION

All the proposed approach is implemented in MATLAB 7.1 software package on a computer with the configuration of Intel i3, 2.13 GHz processor having 2 GB RAM. For the assessment of the developed approach, the system is tested on "Lichtenstein Castle" image acquired from web source <https://en.wikipedia.org/>.

The efficiency of the proposed Spatial Spectral Filtration technique using multivariate thresholding scheme is investigated by performing simulations on the "Lichtenstein Castle" image that is corrupted by Gaussian Noise at different noise levels of $\sigma = 10, 20, 30, 40,$ and 50 as shown in figure (5). For efficient processing, the input image is transformed to size of 128×128 .

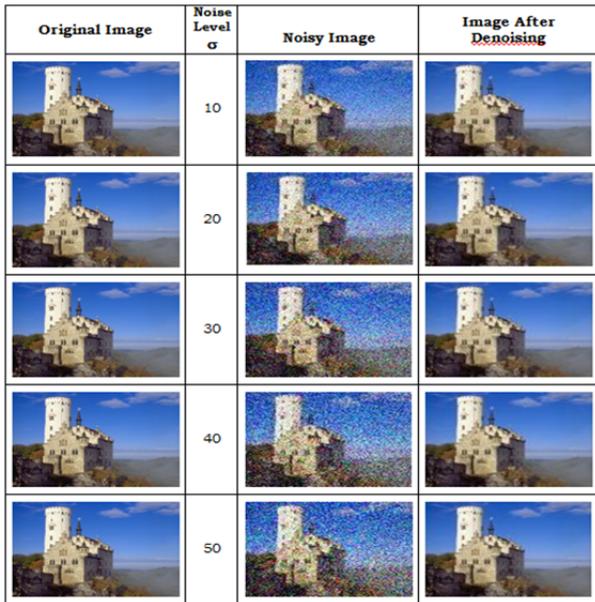


Fig. 5: Denoising of Input Image at different Noise levels by the proposed SSF Method.

We have taken two metrics to evaluate the quality of the denoised image, one is subjective evaluation by the human eyes and the other method is by a quality metric called Peak Signal to Noise Ratio (PSNR) which is computed using the below formulae.

$$PSNR = 10 \log \left[\frac{225^2}{1/(M \times N) \sum_{M=0}^M \sum_{N=0}^N (f(x,y) - \hat{f}(x,y))^2} \right] \quad (8)$$

Where, $f(x,y)$ denotes the original image and $\hat{f}(x,y)$ is the estimated value of the denoised image and $m \times n$ is the resolution of the considered image. The obtained PSNR values of noisy and denoised image at different noise levels for the input image is given in the Table 1. As can be seen from the tabular values, proposed technique is attaining higher PSNR values for the denoised image in addition to maintaining the superior quality of the image.

Table 1: The PSNR values before and after denoising the input image.

For Input Image			
Sl.No	Noise Level σ	Noisy Image PSNR (dB)	Denoised Image PSNR (dB)
1	10	28.17	37.30
2	20	22.09	34.99
3	30	18.60	33.63
4	40	16.05	32.93
5	50	14.18	31.32

It can be established through the achieved objective and subjective results that the developed technique for denoising of images is performing better than the conventional approaches of univariate thresholding based multiwavelet methods and all scalar wavelets based methods.

V. COMPARISON OF VARIOUS DENOISING TECHNIQUES WITH THE PROPOSED SCHEME

Comparison of PSNR values have also been made with various other conventional techniques at different noise levels for the input image and tabulated in table 2. It can be seen that the PSNR values achieved by our technique are much higher as compared with earlier methods. The figure 6 shows the graphical performance comparison of the proposed denoising scheme with earlier conventional methods.

As can be seen from figure 6 the proposed method achieves highest PSNR values among all others. The figure 7 shows the comparative computational time taken to denoise the input image by the conventional wavelet based method and the proposed multiwavelet based multivariate thresholding method.

Table 2: Comparison of various denoising techniques with proposed method at different Noise levels.

Noise level	$\sigma =10$	$\sigma =20$	$\sigma =30$	$\sigma =40$	$\sigma =50$
Noisy Image	28.13	22.16	18.58	16.08	14.18
Complex Shrinkage	26.93	26.71	25.71	23.98	22.17
Bivariate-DWT	34.48	30.44	28.23	26.85	25.71
BLS-GSM	34.80	30.91	28.79	27.40	26.29
GHM Univariate Thersholding	35.08	33.01	31.72	30.85	29.12
Proposed Method	37.30	34.99	33.63	32.93	31.32

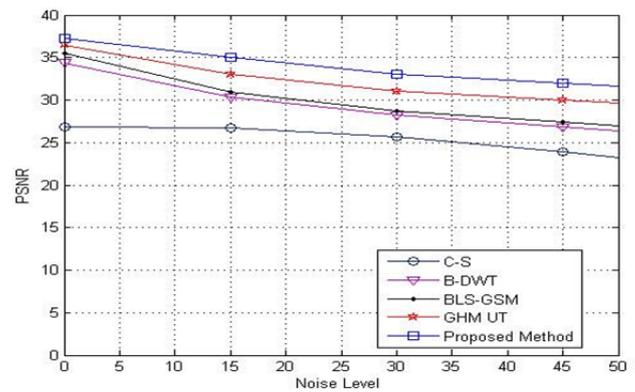


Fig. 6: Comparison of various denoising techniques

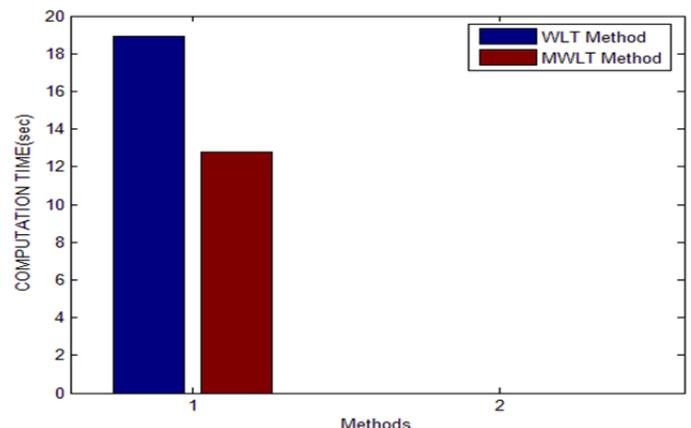


Fig. 7: Comparative Computational Time Taken by Wavelet and Proposed Multiwavelet Based Method for denoising

VI CONCLUSION

In this paper, an enhanced image denoising technique is implemented which is based on multivariate thresholding of correlated coefficients obtained from multiwavelet transformation. The proposed approach termed as Spatial Spectral Filtration is found to obtain improved denoising results as compared with the earlier image denoising approaches. This technique is based on eliminating the noisy components from remote sensing images by decomposing the image through multiwavelets and involves cutting the noisy part from the signal by a process called thresholding. Decomposed coefficients that are highly correlated are taken as components of a vector and a thresholding operation is applied on the whole vector.

The threshold value is carefully computed as the denoising performance on this method greatly depends on it. Several simulations are performed on a number of images to assess the denoising performance of the developed method. The proposed method produces excellent results both objectively and subjectively and the obtained results confirm that this approach is well suited for image denoising applications.

REFERENCES:

1. R. Yang, L. Yin, M. Gabbouj, J. Astola, and Y. Neuvo, "Optimal weighted median filters under structural constraints," *IEEE Transactions on Signal Processing*, Vol. 43, Mar. 1995, pp. 591-604.
2. A. Ben Hamza, P. Luque, J. Martinez, and R. Roman, "Removing noise and preserving details with relaxed median filters," *Journal of Mathematical Imaging and Vision*, Vol. 11, No. 2, Oct. 1999, pp. 161-177.
3. V. Strela. "Denoising via block Wiener filtering in wavelet domain", In *Proceeding of the 3rd European Congress of Mathematics*, Barcelona, July 2000, Birkhauser Verlag.
4. J. Portilla, V. Strela, Martin J. Wainwright, Eero P. Simoncelli, "Adaptive Wiener Denoising using a Gaussian Scale Mixture Model in the wavelet Domain", *Proceedings of the 8th International Conference of Image Processing*, Thessaloniki, Greece. Oct 2001.
5. S. Grace Chang, Bin Yu and Martin Vetterli, "Spatially adaptive wavelet thresholding with context modeling for image denoising", *IEEE Transactions on Image Processing*, Vol. 9, No. 9, Sep 2000, pp.1522-1531.
6. D. Lazzaro and L.B. Montefusco, "Edge-preserving wavelet thresholding for image denoising", *Journal of Computational and Applied Mathematics*. Vol. 210, No. 1-2, 2007, pp. 222-231.
7. M.K.Mihcak, I.Kozintsev, and K.Ramchandran, "Spatially Adaptive Statistical Modeling of Wavelet Image Coefficients and its Application to Denoising", *Proceeding of the IEEE International Conference on Acoustics, Speech, Signal Processing*, Vol.6, 1999, pp. 3253-3256.
8. Kother Mohideen, Arumuga Perumal, Krishnan, Mohamed Sathik, "Image Denoising And Enhancement Using Multiwavelet With Hard Threshold In Digital Mammographic Images", *International Arab Journal of e-Technology*, Vol. 2, No. 1, January 2011, pp. 49-55.
9. Prayoth Kumsawat, Kitti Attakitmongcol and Arthit Srikaew, "A Robust Image Watermarking Scheme Using Multiwavelet Tree", *Proc. of the World Congress on Engineering*, London, U.K., Vol 1, 2007.
10. L. W. She and B. Zheng, "Multiwavelets based denoising of SAR image", *Proceeding of the 5th International Conference on Signal Processing*, Beijing, China, Vol. 1, 2000, pp. 321-324.
11. V. Strela and A. T. Walden, "Signal and image denoising via wavelet thresholding: orthogonal, biorthogonal, scalar and multiple wavelet transforms", in *Nonlinear and Nonstationary Signal Processing*, W. F. Fitzgerald, R. L. Smith, A. T. Walden, and P. C. Young, Eds., Cambridge University Press, Cambridge, UK, 2001, pp. 395-441.
12. Chen G.Y and Bui T. D, "Multi-wavelet Denoising using Neighboring Coefficients", *IEEE Signal Processing Letters*, Vol. 10, No. 7, 2003, pp. 211- 214.
13. Gijbels, Irene, Alexandre Lambert, and Peihua Qiu. "Edge-preserving image denoising and estimation of discontinuous surfaces", *IEEE Transactions on Pattern Analysis and Machine Intelligence*, Vol. 28, No.7, 2006, pp. 1075-1087.
14. S. Grace Chang, Bin Yu and M. Vattereli, "Adaptive Wavelet Thresholding for Image Denoising and Compression", *IEEE Trans. on Image Processing*, Vol. 9, No. 9, Sept. 2000, pp. 1532-1546.
15. S.Mallat, "A Wavelet Tour of Signal Processing", Academic Press, 1998, San Diego, California, USA.
16. D. L. Donoho and I. M. Johnstone, "Ideal spatial adaptation via wavelet shrinkage", *Biometrika*, Vol. 81, 1994, pp. 425-455.
17. T. R. Downie and B. W. Silverman, "The discrete multiple wavelet transform and thresholding methods," *IEEE Transactions on Signal Processing*, Vol. 46, No. 9, 1998, pp. 2558-2561.
18. Md Ateeq Ur Rahman and Shaik Rusthum, "High Resolution Data Processing for Spatial Image Data Mining", *International Journal Of Geomatics And Geosciences*, Vol 1, No 3, December 2010, pp. 327-342.
19. M. B. Martin and A. E. Bell, "New image compression techniques using multiwavelets and multiwavelet packets," *IEEE Transactions on Image Processing*, Vol. 10, No. 4, 2001, pp. 500-511.
20. Md Ateeq ur Rahman, Shaik Rusthum and I.V Murali Krishna, "Pattern Generation from Spatial Database for the Automated Interpretation of Remotely Sensed Imagery", *International Journal of Remote Sensing & Geoscience*, Vol. 2, Issue 5, Sept. 2013, pp. 33-41.